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COMMENT

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Abstract. In a paper recently published in this journal by Gordeyev, a simultaneous expansion of the electromagnetic field of a point charge in powers of the acceleration and its time derivatives has been given. Since the derivation is rather involved it seems worthwhile to point out that results of this kind may be established in a much simpler way.

We start from the retarded Liénard–Wiechert potentials in the form

$$\phi(\boldsymbol{x}, t) = 2e \int \theta(t-t')\, \delta[|\boldsymbol{x}-\boldsymbol{x}(t')|^2-(t-t')^2]\, \mathrm{d}t' \tag{1}$$

and

$$\boldsymbol{A}(\boldsymbol{x}, t) = 2e \int \theta(t-t')\, \delta[|\boldsymbol{x}-\boldsymbol{x}(t')|^2-(t-t')^2]\boldsymbol{v}(t')\, \mathrm{d}t' \tag{2}$$

where $\boldsymbol{x}(t')$ is the space trajectory of the charge and $\boldsymbol{v}(t')$ its velocity. Defining $R\boldsymbol{n} \equiv \boldsymbol{x}-\boldsymbol{x}(t)$ as the distance from the field point $\boldsymbol{x}$ to the simultaneous position $\boldsymbol{x}(t)$ of the charge, we may write

$$\boldsymbol{x}-\boldsymbol{x}(t') = R\boldsymbol{n}+\boldsymbol{x}(t)-\boldsymbol{x}(t-\tau) \tag{3}$$

where $\tau \equiv t-t'$. A Taylor expansion gives

$$\boldsymbol{x}-\boldsymbol{x}(t') = R\boldsymbol{n}+\tau\boldsymbol{v}+\tau\boldsymbol{g}(\tau) \tag{4}$$

where

$$\boldsymbol{g}(\tau) \equiv \sum_{l=1}^{\infty} \frac{(-1)^l}{(l+1)!}\tau^l \overset{l}{\boldsymbol{v}} \tag{5}$$

is a function of the accelerations $\boldsymbol{a} \equiv \overset{1}{\boldsymbol{v}}$, $\boldsymbol{a} \equiv \overset{2}{\boldsymbol{v}}$, etc. Substituting (4) into (1) and expanding the δ-function around $\boldsymbol{g}=0$, we find

$$\phi(\boldsymbol{x}, t) = 2e \int \theta(\tau)\, \mathrm{e}^{\boldsymbol{g}(\tau)\cdot\partial/\partial\boldsymbol{v}}\, \delta[|R\boldsymbol{n}+\tau\boldsymbol{v}|^2-\tau^2]\, \mathrm{d}\tau. \tag{6}$$

Inserting (5) into (6) we get an infinite product of exponentials. We write each exponential as a power series and perform the time integration. We then arrive at

$$\phi(\boldsymbol{x}, t) = e \sum_{n_1, n_2, \ldots} \frac{(-1)^m R^{m-1}}{n_1!n_2!\ldots} \frac{\partial^n}{\partial \boldsymbol{v}^n}\Phi_{m-1}\left(\frac{\overset{1}{\boldsymbol{v}}}{2!}\right)^{n_1}\left(\frac{\overset{2}{\boldsymbol{v}}}{3!}\right)^{n_2}\ldots \tag{7}$$

where $n \equiv \Sigma n_l$, $m \equiv \Sigma l n_l$ and where Φ_{m-1} is the function defined by Gordeyev in formula (21) of her paper. The expansion for the vector potential may be established in a similar way.

In expression (7) one may order the summations according to increasing values of m. One then obtains Gordeyev's result (18,22). A second possibility is to order according to increasing values of n. The latter seems the more approriate one for the purpose of obtaining a perturbation expansion in terms of $\alpha \equiv e^2/mR_{\min}$.

It may be mentioned that simultaneous expansions linear in the accelerations have already been considered by Schott (1912, Appendix C).

References

Gordeyev A N 1975 *J. Phys. A: Math. Gen.* **8** 1048–59
Schott G A 1912 *Electromagnetic Radiation* (Cambridge: University Press)